

## Solution Set 6 (Fall 2011)

### 6.1

$$C=10\mu\text{f}$$

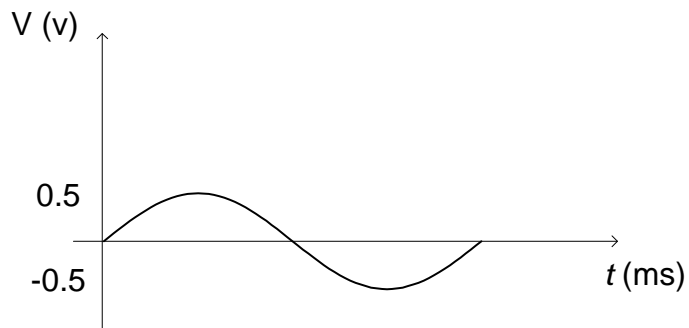
$$i(t)=10\cos 10^3 t \text{ (mA)} = (10\text{m}).\cos 10^3 t \text{ (A)}$$

$$i(t<0)=0, \quad v(t<0)=0$$

(a) Find the expression for the voltage across the capacitors.

$$\begin{aligned} V &= \frac{1}{C} \int_0^t i dt = \frac{1}{10\mu} \int_0^t (10\text{m}) \cos 10^3 t dt \\ &= \frac{10\text{m}}{10\mu} \cdot \frac{1}{10^3} \sin 10^3 t \\ &= \sin 10^3 t \text{ (v)} \end{aligned}$$

(b) sketch the voltage across the capacitor:



(c) Find the expression for power.

$$\begin{aligned} P &= i \cdot v = (5\text{m}).\cos 10^3 t * 0.5 \sin 10^3 t \\ &= 1.25 \sin(2 \cdot 10^3)t \text{ (mW)} \end{aligned}$$

## 6.2

$$i = 2 \text{ mA} \cdot \frac{dv}{dt} = \frac{5v}{10v}$$

Find c:

$$i = c \frac{dv}{dt}$$

$$2 \text{ mA} = c \cdot \frac{5}{10}$$

$$C = 4 \text{ mF}$$

## 6.3

$$C = 2 \text{ } \mu\text{F}$$

$$i = c \frac{dv}{dt}$$

$$0 < t < 6 \text{ ms}, \frac{dv}{dt} = \frac{12}{6 \text{ ms}} = 2 \text{ k}$$

$$i = c \frac{dv}{dt} = (2 \mu) \cdot (2 \text{ k}) = 4 \text{ mA}$$

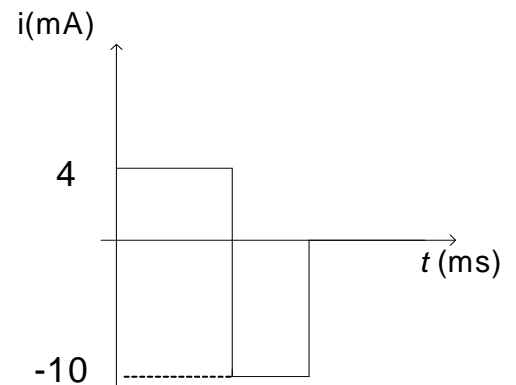
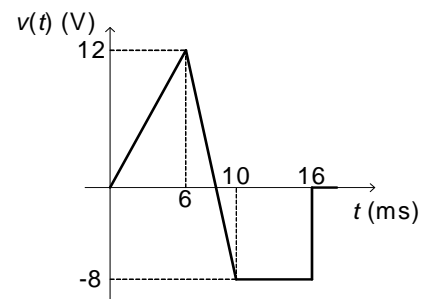
$$6 \text{ ms} < t < 10 \text{ ms}, \frac{dv}{dt} = \frac{-20}{4 \text{ ms}} = -5 \text{ k}$$

$$i = c \frac{dv}{dt} = (2 \mu) \cdot (-5 \text{ k})$$

$$= -10 \text{ mA}$$

$$10 \text{ ms} < t < 16 \text{ ms}, \frac{dv}{dt} = 0$$

$$i = c \frac{dv}{dt} = 0$$



## 6.4

$$C = 10\mu\text{F}$$

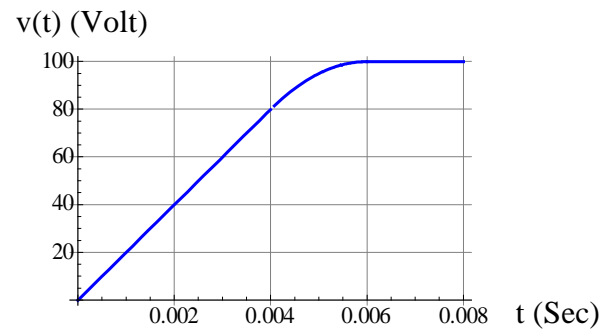
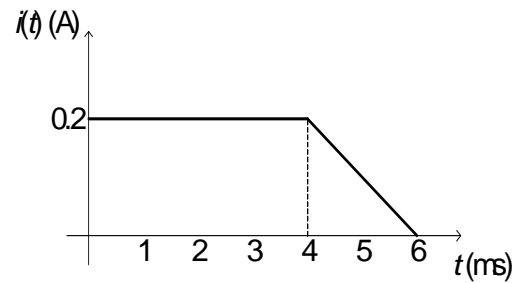
$$v_o = -0.1 \text{ V}$$

$$v(t_2) - v(t_1) = \frac{1}{C} \int_{t_1}^{t_2} i dt$$

$$0 < t < 4 \text{ ms}, \quad v(t) = v_o = -0.1 \text{ V}$$

$$i = 0.2 \text{ A}$$

$$\begin{aligned} v &= \frac{1}{10\mu} \int_0^t (0.2) dt + (-0.1) \\ &= (20 * 10^3) t - 0.1 \text{ (V)} \end{aligned}$$



$$4 \text{ ms} < t < 6 \text{ ms}, \quad v(t_1) = (20 * 10^3) (4 \text{ ms}) - 0.1$$

$$= 0.8 - 0.1 = 79.9 \text{ V}$$

$$i = -0.1 \text{ A}$$

$$v = \frac{1}{10\mu} \int_{4 \text{ ms}}^t (-100t + 0.6) dt + 79.9$$

$$= 10^5 \left[ \frac{-100t^2}{2} + 0.6t \right]_{4 \text{ ms}}^t + 79.9$$

$$= (5 * 10^6) t^2 + 60 * 10^3 t - 80.1 \text{ (V)}$$

$$t \geq 6 \text{ ms}, \quad v(t_1 = 6 \text{ ms}) = (5 * 10^6) (6 * 10^{-3})^2 + 60 * 10^3 (6 * 10^{-3}) - 80.1 = 99.9 \text{ (V)}$$

## 6.5

$$\frac{di}{dt} = \frac{100m}{2m}, \quad v = 100 \text{ mV}$$

Find L:

$$v = L \frac{di}{dt}$$

$$100m = L \cdot \frac{100m}{2m}$$

$$L = 2 \text{ mH}$$

## 6.6

$$L = 10 \text{ mH}$$

$$v = L \frac{di}{dt}$$

$$0 < t < 2 \text{ ms} : \frac{di}{dt} = -5$$

$$v = L \frac{di}{dt} = -50 \text{ mV}$$

$$0 < t < 4 \text{ ms} : \frac{di}{dt} = 0$$

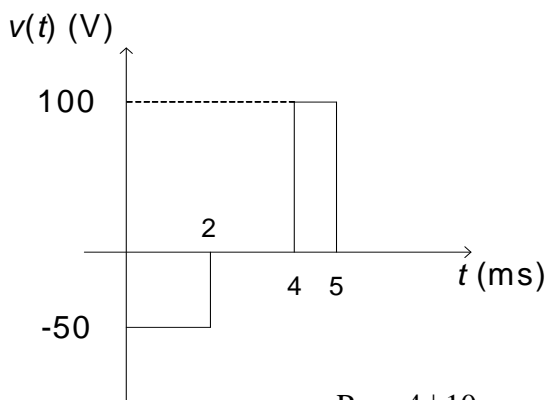
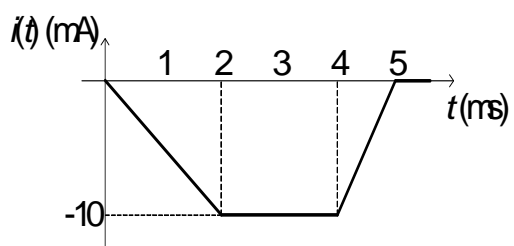
$$v = 0$$

$$0 < t < 5 \text{ ms} : \frac{di}{dt} = 10$$

$$v = L \frac{di}{dt} = 10 \text{ m} \cdot 10 = 100 \text{ mV}$$

$$t > 5 \text{ ms} : \frac{di}{dt} = 0$$

$$v = 0$$



## 6.7

$$i_o = 0 \text{ mA}$$

$$L = 20 \text{ mH}$$

$$i(t_2) - i(t_1) = \frac{1}{L} \int_{t_1}^{t_2} v dt$$

$$0 < t < 2 \text{ ms} : v = 20 \text{ m}$$

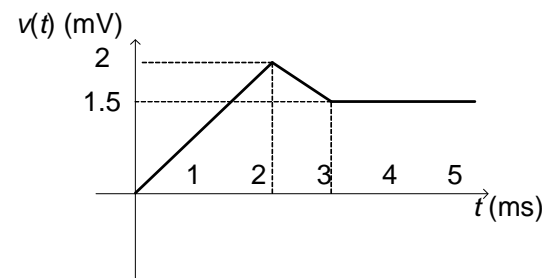
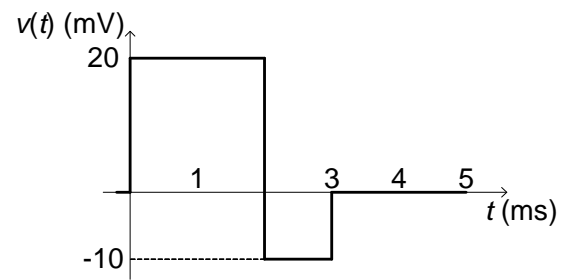
$$i(t_1) = i_0 = 0$$

$$\begin{aligned} i &= \frac{1}{L} \int_0^t (20 \text{ m}) dt \\ &= \frac{1}{20 \text{ m}} \cdot (20 \text{ m} \cdot t) \\ &= t \end{aligned}$$

$$2 < t < 3 \text{ ms} : v = -10 \text{ m}$$

$$i(t_1) = 2 \text{ m}$$

$$\begin{aligned} I &= \frac{1}{L} \int_{2 \text{ m}}^t (-10 \text{ m}) dt + 2 \text{ m} \\ &= \frac{1}{20 \text{ m}} (-10) (t - 2 \text{ m}) + 2 \text{ m} \\ &= \frac{1}{20 \text{ m}} (20 \text{ m}^2 - 10 \text{ m} t) + 2 \text{ m} \end{aligned}$$



$$= 3\text{m} - \frac{t}{2}$$

$$3\text{ ms} < t < \infty : v = 0$$

$$i(t_1) = 3\text{m} - \frac{3\text{m}}{2} = 1.5\text{m}$$

$$i = i(t_1) + 0 = 1.5\text{ m}$$

## 6.8

$$I_o = -0.4\text{ A}$$

$$L = 10\text{ mH}$$

$$i(t_2) - i(t_1) = \frac{1}{L} \int_{t_1}^{t_2} v dt$$

$$0 < t < 1\text{ms} : i(t_1) = i_o = -0.4\text{ A}$$

$$v = \frac{10}{1\text{m}} t = (10\text{k}) t$$

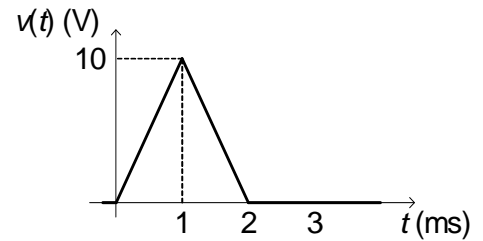
$$i = \frac{1}{L} \int_0^t v dt + i(t_1)$$

$$= \frac{1}{10\text{m}} (10\text{k}) \frac{t^2}{2} - 0.4$$

$$= (0.5 * 10^6) t^2 - 0.4 \text{ (A)}$$

$$1\text{ms} < t < 2\text{ms} : i(t_1) = (0.5 * 10^6) (1\text{m})^2 - 0.4 = 0.1\text{ A}$$

$$v = (-10\text{k})t + 20$$



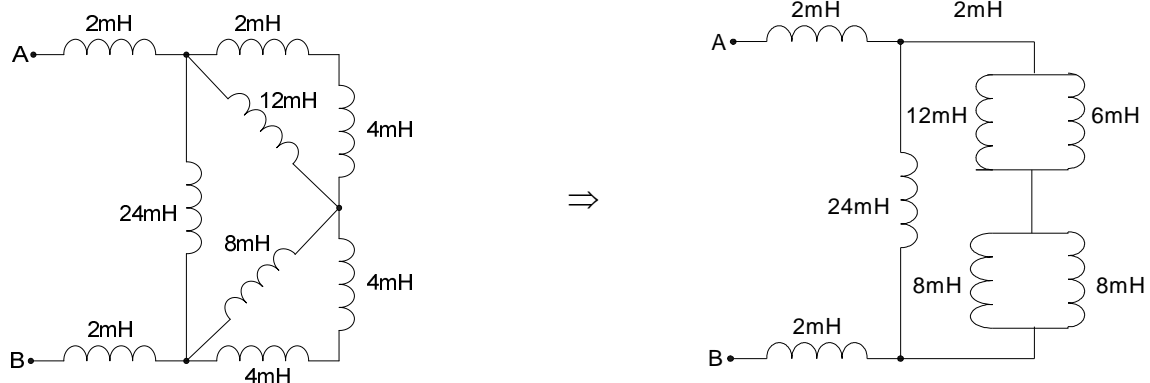
$$\begin{aligned}
 i &= \frac{1}{L} \int_{1m}^t [(-10k)t + 20] dt + 0.1 \\
 &= \frac{1}{10m} [(-10k) \frac{t^2 - (1m)^2}{2} + 20(t - 1m)] + 0.1 \\
 &= (-500k)t^2 + (2k)t - 1.4
 \end{aligned}$$

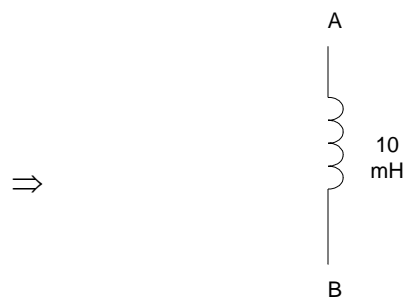
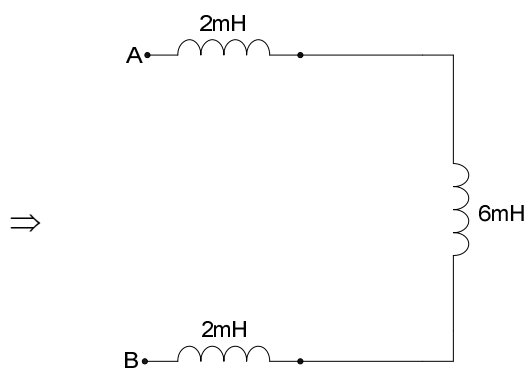
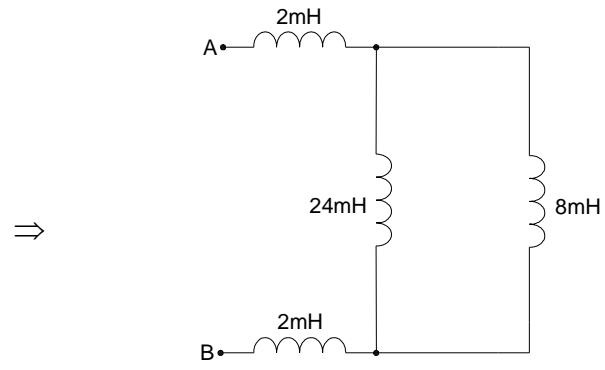
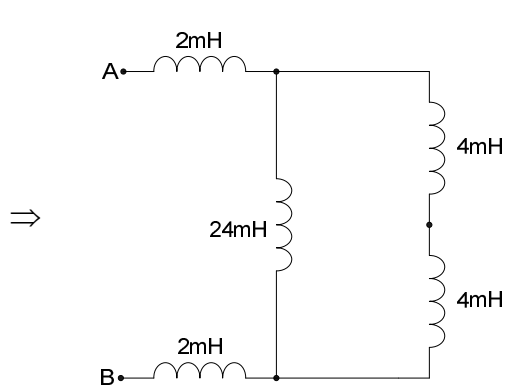
$$\begin{aligned}
 2ms < t < \infty : i(t_1) &= (-500k)(2m)^2 + (2k)(2m) - 1.4 \\
 &= -2 + 4 - 1.4 \\
 &= 0.6 \text{ (A)}
 \end{aligned}$$

\*Note: When apply equations  $v(t_2) - v(t_1) = \frac{1}{C} \int_{t_1}^{t_2} i dt$  ,  $i(t_2) - i(t_1) = \frac{1}{L} \int_{t_1}^{t_2} v dt$

keep in mind that  $v(t_1)$  and  $i(t_1)$  refer to initial conditions at  $t_1$  .  $v(t_2)$  and  $i(t_2)$  are functions of  $t$  .Not a specific value at time  $t_2$  . Therefore, in the integral ,the upper limit  $t_2$  should be a variable of  $t$  .

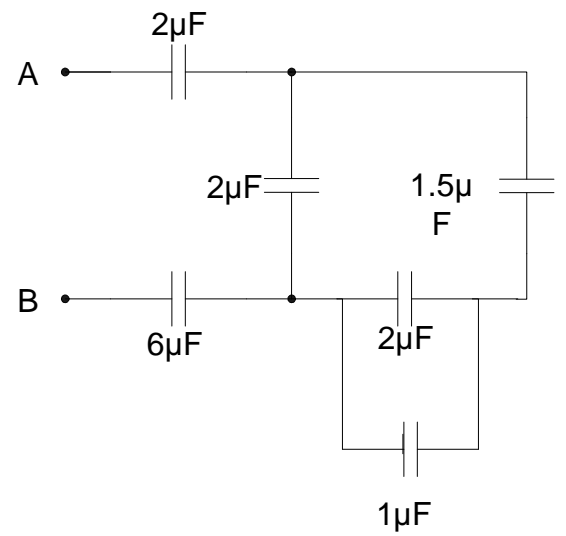
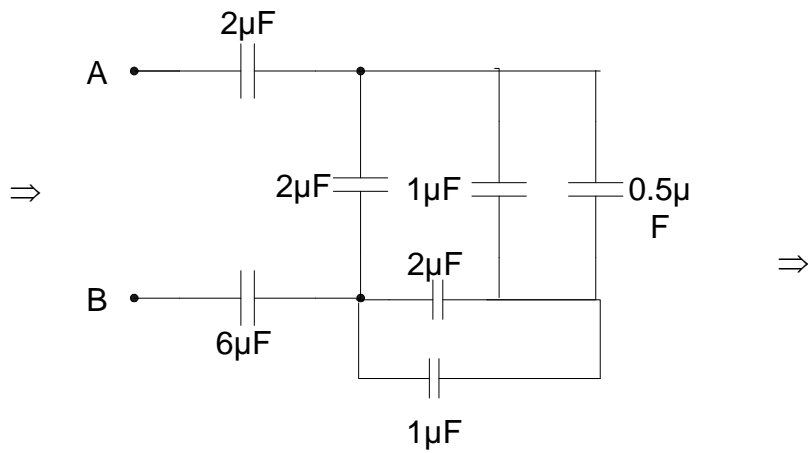
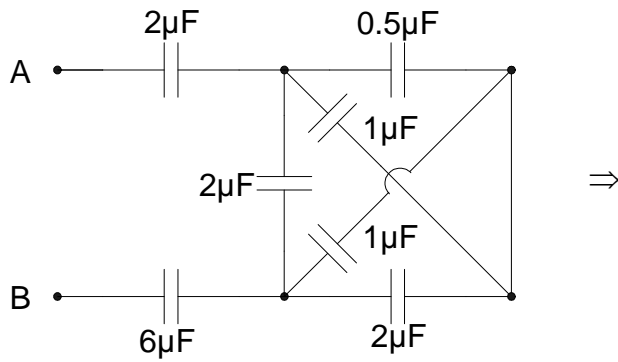
## 6.9

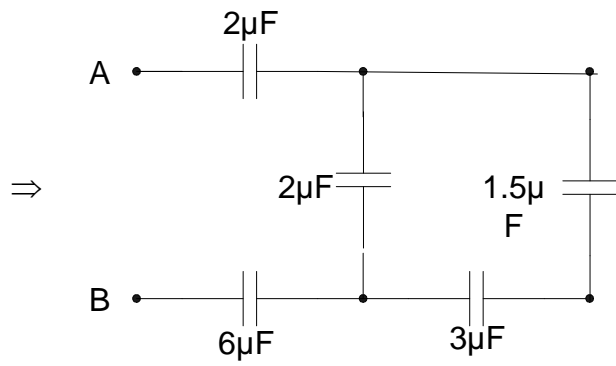




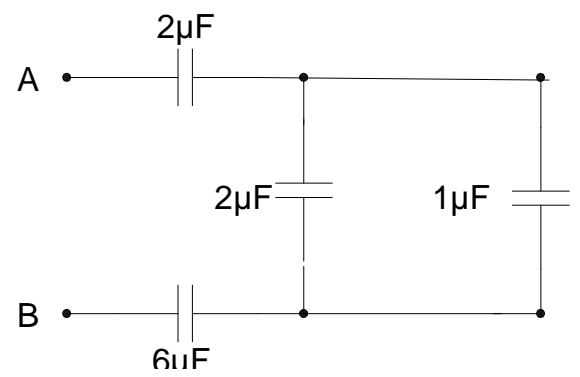


## 6.10

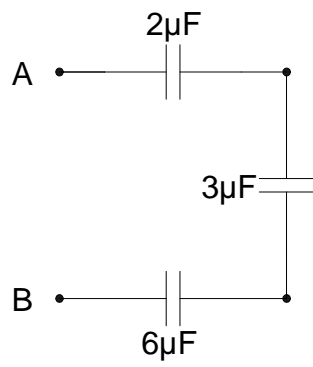




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